Math 2050, HW 6

Q1. Suppose $f : [0, 1] \to \mathbb{R}$ is a continuous function such that $f([0, 1]) \subset \mathbb{Q}$. Show that f is a constant function.

Q2. Let $f : [0, \frac{\pi}{2}] \to \mathbb{R}$ is a function given by $f(x) = \sup\{x^2, \cos x\}$. Show that there exists an absolute minimum point $x_0 \in [0, \frac{\pi}{2}]$ for f. Moreover, show that x_0 is the solution to $x^2 = \cos x$.

Q3. Show that the function $f(x) = x^{-2}$ is uniformly continuous on $[1, +\infty)$ but is not on $(0, +\infty)$.

Q4. Suppose $f, g : A \to \mathbb{R}$ are uniformly continuous and is bounded on A, show that fg is also uniformly continuous. Is the same assertion true without the boundedness assumption? Justify your answer.

Q5. Show that if f is continuous on $[0, +\infty)$ and is uniformly continuous on $[a, +\infty)$ for some $a > 0$, then f is uniformly continuous on $[0, +\infty)$.